Dynamic optimisation of ecological systems with hysteretic dynamics and stochastic thresholds

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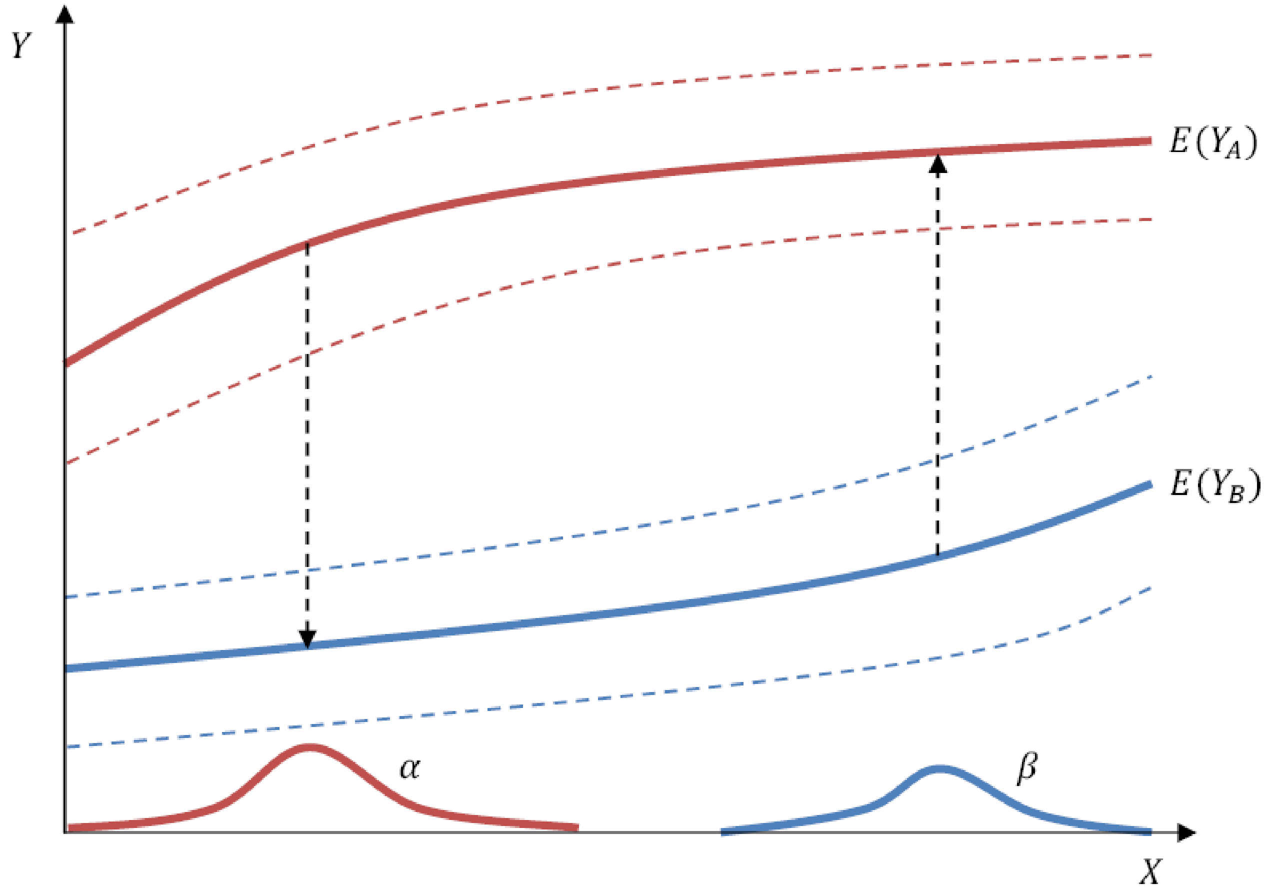
Motivation

- Initial inspiration came from Nævdal and Oppenheimer (2007)
  - Irreversible regime shift
  - Optimal control framework
  - Assumes unknown (but fixed) threshold location
  - Partition state space into ‘safe’ and ‘unsafe’ regions
- What if the regime shift is reversible but the system has hysteretic dynamics?
- What if the threshold location is not fixed but stochastic?
- Is there a more general model that is suitable for many different types of ecosystem dynamics, different assumptions about threshold location and explicitly considers learning (resolution of uncertainty) over time?

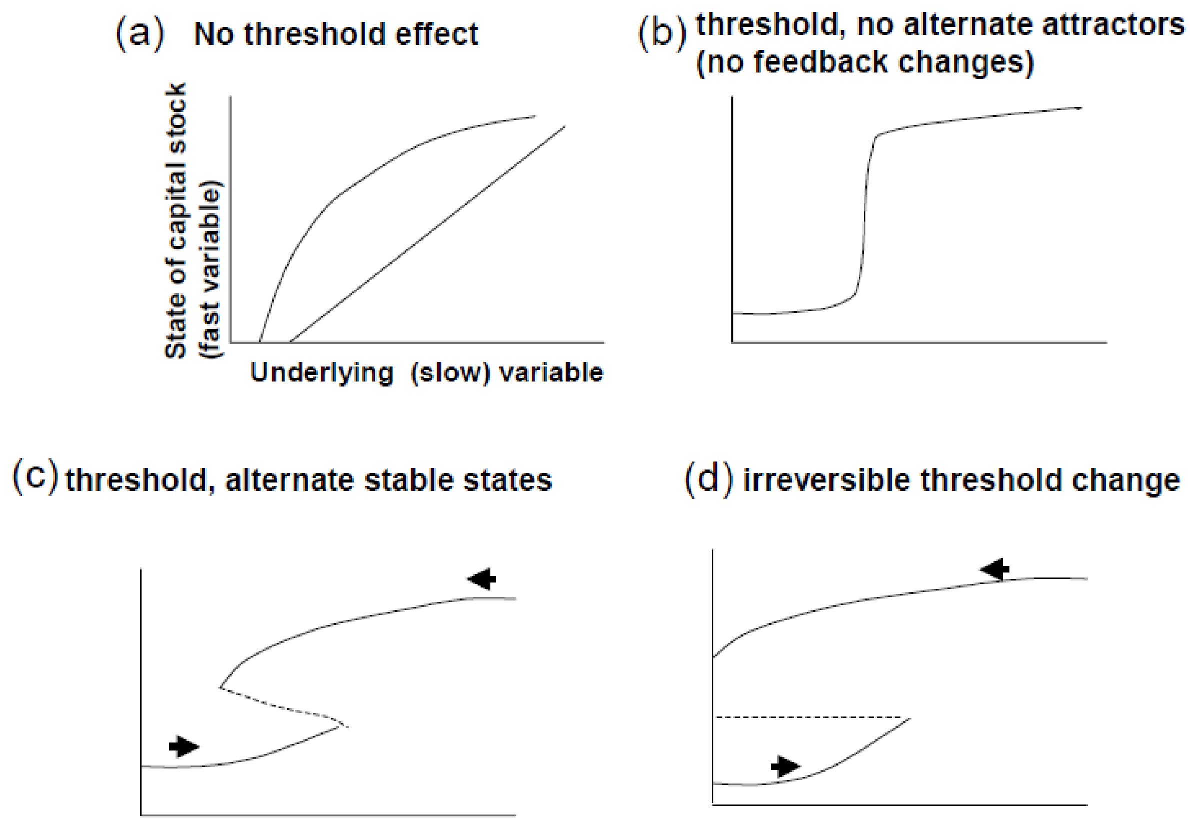
A system with two alternative regimes and hysteretic dynamics



General nature of the model’s application

- Hysteretic dynamics
- Irreversibilities
- Continuous dynamics with a threshold effect
- Stochastic or fixed threshold location
- Safe minimum standards (SMS): ‘safe’ and ‘unsafe’ sub-spaces

Different types of ecosystem dynamics



Source: Walker *et al.* (2010)

Potential applications

- Fish population (Y) as a function of water quality (X)
- Koala population (Y) as a function of habitat size or connectedness (X)
- Crop yield (Y) as a function of groundwater table depth (X)
- Prey species population (Y) as a function of predator species population (X)
- Potentially two sources of utility from the single output
  - Direct (from Y)
  - Consumption (from q)

The objective of the ‘manager’

- Assume an ‘ecosystem manager’
- Objective: maximise NPV of expected utility from the system over a finite time horizon [1,T]
  - Expectation formed over all possible contingencies
  - Weightings based on current level of information about system

$$V_A = \max_{c_0', q_A, q_B} \left\{ \alpha_0(X_0')[e^{-\rho} U(Y_A(X_0'), q_A) + V_{AA}] + \left(1 - \alpha_0(X_0')\right)[e^{-\rho} U(Y_B(X_0'), q_B) + V_{AB}] \right\}$$

- Time-invariant instantaneous utility $U(Y_t, q_t)$
- Assume that correct incentive structure is in place
  - Ignore principal-agent, political economy problems

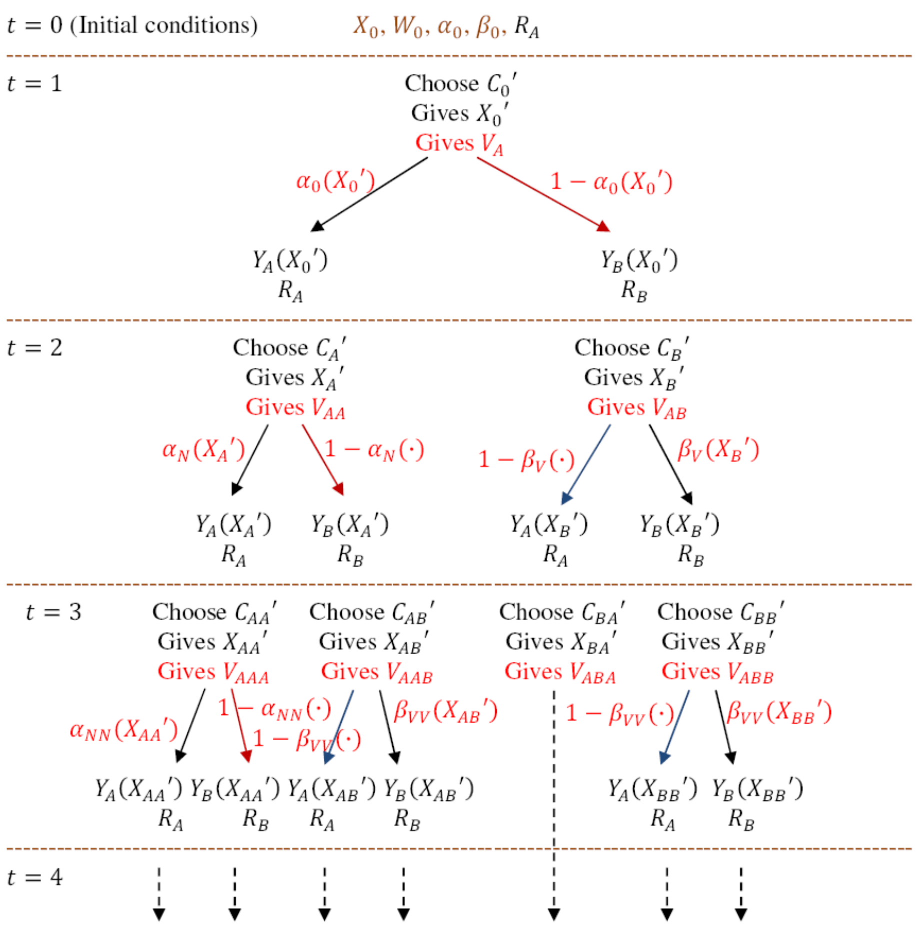
The objective of the 'manager'

- The objective of the manager is to choose the level of investment (*C*) in controlling *X* (in the next period only) that maximises the expected NPV of utility from the system
- This expected NPV is conditional on the current level of information about the system, which is continually updated after every time period based on every (hypothetical) contingency/outcome
- After every period, a signal is provided and more is learnt about the system's dynamics

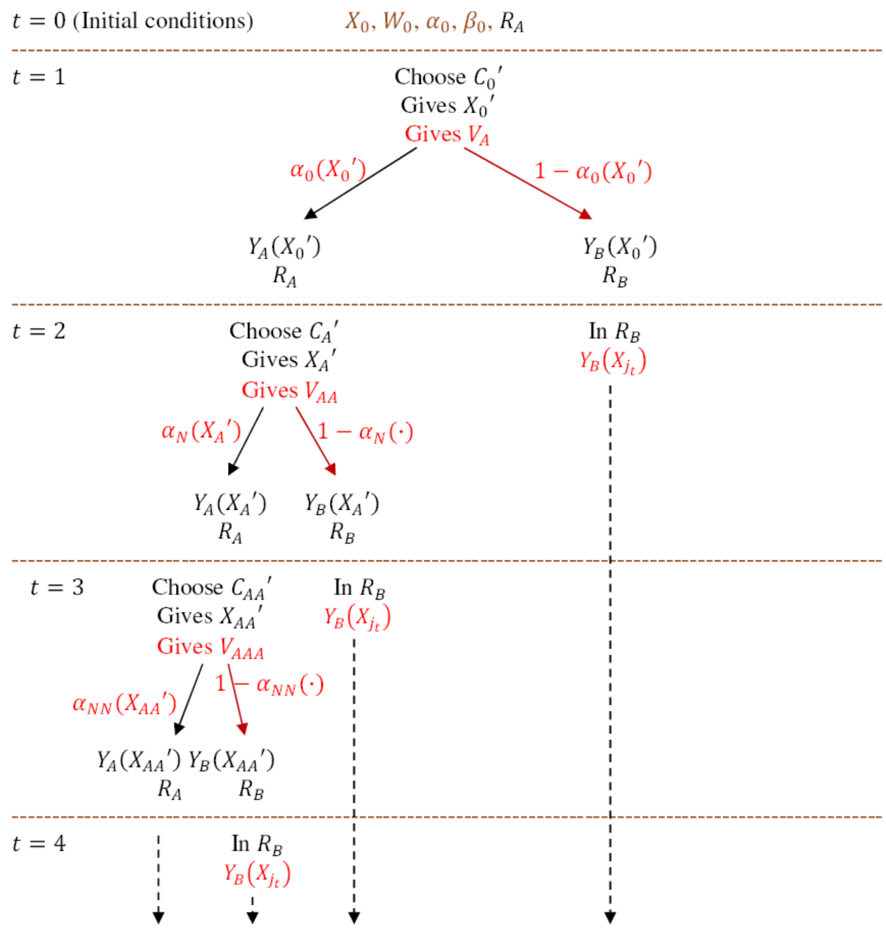
Model characteristics

- Two alternative system regimes (R_A and R_B)
- Hysteretic dynamics
- Stochastic threshold locations (α and β)
- Stochastic production functions (Y_A and Y_B)
  - Single input, single output
  - Output is regime-contingent
- Learning (about locations of Threshold A and Threshold B)

The optimisation for a system with hysteretic dynamics



The optimisation for a system with an irreversibility



Updating rules for α and β

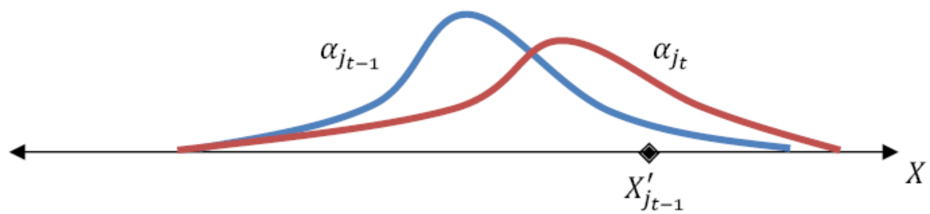
- Prior probability distribution postulated based on historical data, experimental data and/or export knowledge
- If the system begins the time period in Regime A (B) then only the α- (β-) PDF can be updated
- Different assumptions about the threshold will result in different updating rules
  - stochastic or fixed
- Flexibility in choice of updating rule
  - Bayesian updating, multiple priors, minimax, etc.

Stochastic threshold

$$\alpha_{j_t} \equiv \alpha^{L}_{j_{t-1}} = f\left(X'_{j_t} \middle| \alpha_{j_{t-1}}, X'_{j_{t-1}}, F_{j_{t-1}}\right)$$

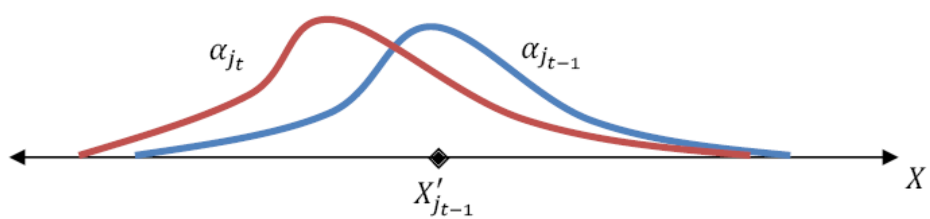
Scenario 1 (Threshold A is crossed)

$F_{j_{t-1}} = 1$



Scenario 2 (Threshold A is <u>not</u> crossed)

$F_{j_{t-1}} = 0$

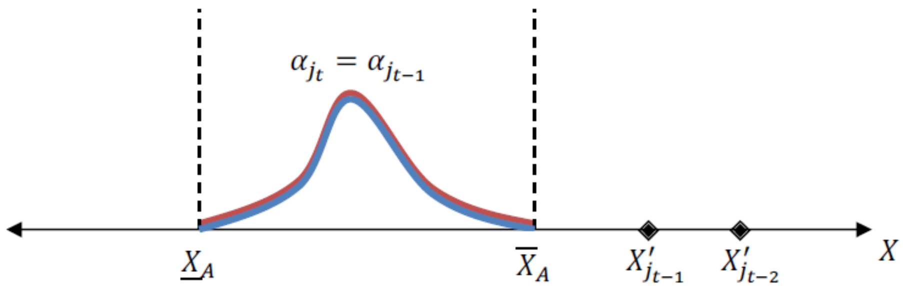


Fixed threshold

$$\alpha_{j_t} \equiv \alpha^L_{j_{t-1}} = f\left(X'_{j_t} \middle| \alpha_{j_{t-1}}, X'_{j_{t-1}}, \overline{X}_A, \underline{X}_A, F_{j_{t-1}}\right)$$

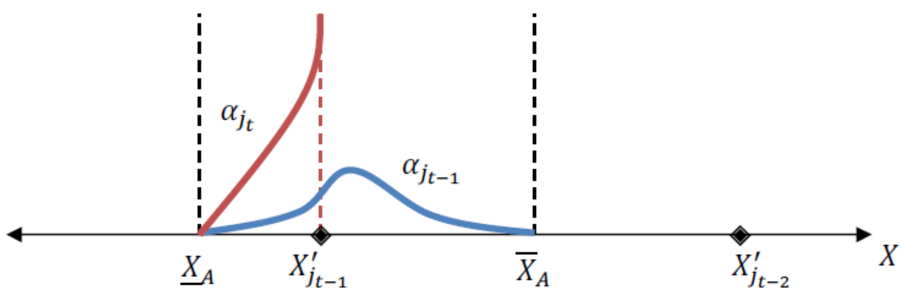
Scenario 1 (SMS threshold is ***not*** *crossed)*

$F_{j_{t-1}} = 0$



Scenario 2 (SMS threshold is crossed but system threshold is ***not*** *crossed)*

$F_{j_{t-1}} = 0$

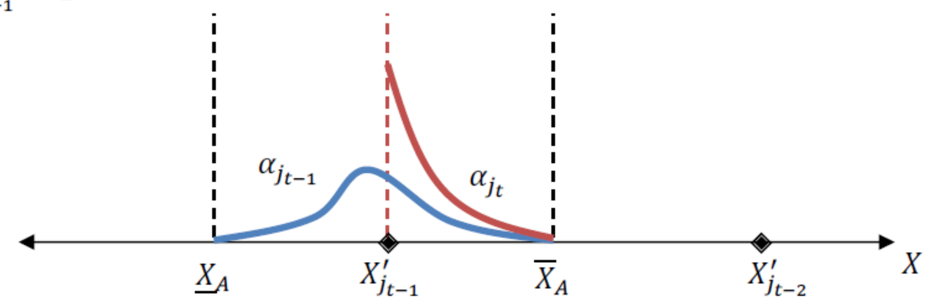


Fixed threshold

$$\alpha_{j_t} \equiv \alpha^{L}_{j_{t-1}} = f\left(X'_{j_t} \middle| \alpha_{j_{t-1}}, X'_{j_{t-1}}, \overline{X}_A, \underline{X}_A, F_{j_{t-1}}\right)$$

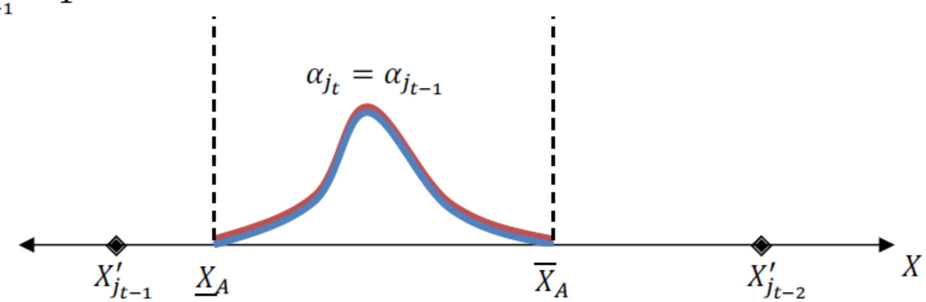
Scenario 3 (SMS threshold is crossed and system threshold is crossed)

$F_{j_{t-1}} = 1$



Scenario 4 (maximum perturbation threshold is crossed)

$F_{j_{t-1}} = 1$



The value function

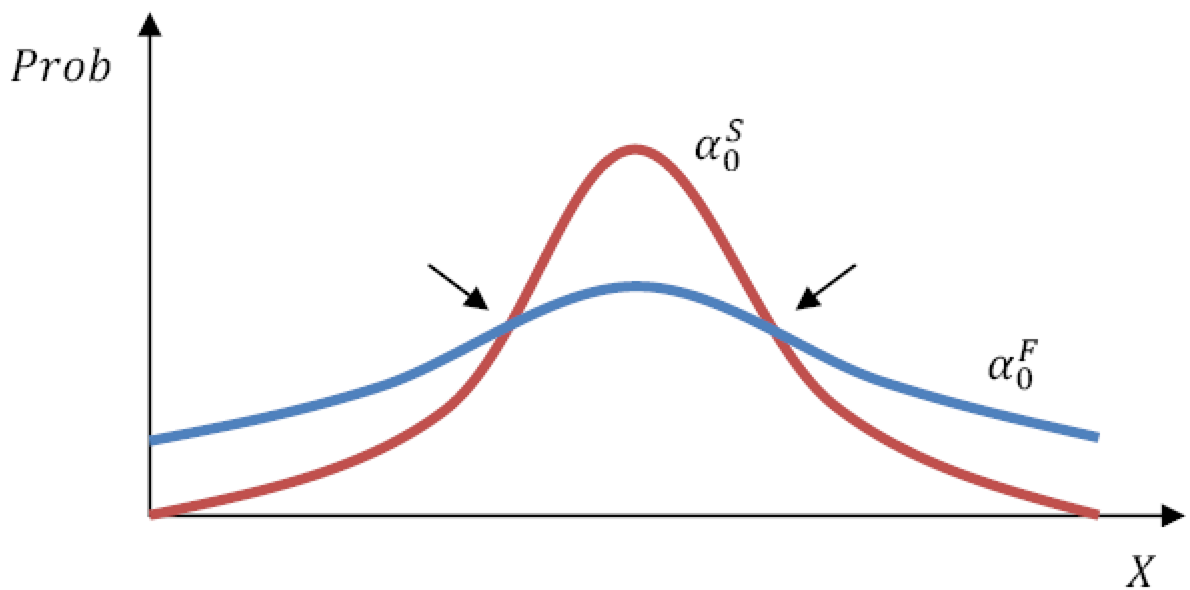
$$V_{j_{t(i_t)}}\left(\boldsymbol{C}'_{\boldsymbol{j_t}}\middle|X'_{j_{t-1}},W_{j_{t-1(i_t)}}\right)$$

$$= \max_{\boldsymbol{C}'_{\boldsymbol{j_t}},q_{j_{t(A)}},q_{j_{t(B)}}} \left\{ \begin{array}{ll} \begin{array}{c} e^{-\rho t}EU_{j_{t(A)}} + \\ \alpha_{j_{t-1}}(X'_{j_t})\left[V_{j_{t+1(A)}}\left(\boldsymbol{C}'_{\boldsymbol{j_{t+1}}}\middle|X'_{j_t},W_{j_{t(A)}}\right)\right] + \\ \left[1-\alpha_{j_{t-1}}(X'_{j_t})\right]\left[V_{j_{t+1(B)}}\left(\boldsymbol{C}'_{\boldsymbol{j_{t+1}}}\middle|X'_{j_t},W_{j_{t(B)}}\right)\right] \end{array} & if\ i_t = A \\ \\ \begin{array}{c} e^{-\rho t}EU_{j_{t(B)}} + \\ \beta_{j_{t-1}}(X'_{j_t})\left[V_{j_{t+1(B)}}\left(\boldsymbol{C}'_{\boldsymbol{j_{t+1}}}\middle|X'_{j_t},W_{j_{t(B)}}\right)\right] + \\ \left[1-\beta_{j_{t-1}}(X'_{j_t})\right]\left[V_{j_{t+1(A)}}\left(\boldsymbol{C}'_{\boldsymbol{j_{t+1}}}\middle|X'_{j_t},W_{j_{t(A)}}\right)\right] \end{array} & if\ i_t = B \end{array} \right\}$$

Hypotheses

1. a. For a stochastic threshold:

Lower uncertainty about the location of Threshold A (i.e. prior α-PDF with lower variance) will result in greater investment if X is close to the mode of the distribution but lower investment if X is far from the mode of the distribution



Hypotheses

1. b. For a fixed threshold:

- Can only learn more about the location of the threshold by crossing the SMS threshold and entering the ‘risky’ sub-space
  - Different to a stochastic threshold, where more is learnt after every period

- For a small perturbation of X past the SMS threshold:
  - ‘Low’ probability of crossing the threshold and learning its location
  - If do cross the threshold then have a small/narrow interval for its exact location

- For a large perturbation of X past the SMS threshold:
  - ‘High probability of crossing the threshold and learning its location
  - If do cross the threshold then have a large/wide interval for its exact location

Hypotheses

1. b. For a fixed threshold:

- Greater uncertainty (i.e. higher variance) in α-PDF
  - Willing to accept a higher probability of crossing the threshold in order to resolve some of this uncertainty

- Longer time horizon:
  - Willing to accept a higher probability of crossing the threshold in order to resolve some of the uncertainty in α-PDF because utility/output losses incurred in the near term may be more than compensated for by requiring lower amounts of investment in the longer term (if the discount rate isn't prohibitively high)

Hypotheses

2. If the system is currently in Regime A, lower uncertainty about the location of Threshold B (i.e. prior β-PDF with lower variance) will result in lower investment in controlling X

   – The manager will be willing to accept a higher probability of crossing Threshold A because there is less uncertainty about the cost of recovering to Regime A

Hypotheses

3. A greater relative contribution of direct utility (from Y) to total utility will result in a greater level of investment in controlling the system

   – Consumption/utility smoothing
   – Maintaining fairly constant U(Y) requires maintaining fairly constant Y
   – Achieved by ensuring a low probability of shifting from Regime A to Regime B
   – Greater investment in controlling X

Hypotheses

4. For a higher discount rate, investment will be lower

   – Higher discount rate means lower weighting on utility received later in the time horizon
   – Less incentive to invest in and maintain the system
   – More incentive to mine the system and live off the wealth acquired in early time periods

Hypotheses

5. For higher initial wealth, investment in controlling X will be lower

   - For higher initial wealth, the relative contribution of output to NPV of utility will be lower
   - Negative consequences of taking greater risks are not as significant
   - Accept higher probability of regime shift by investing less to control the system